Rarefied Flow through a Packed Bed of Spheres

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Abstract. In this paper we present experimental results for flows through packed bed of spheres, in cases where the flow characteristics can be classified in a range from the velocity slip regime to the transition and free molecular flows. A simple simulation model is also proposed and solved by means of a MonteCarlo code, and comparisons between results of numerical calculations and experimental data are also reported.

INTRODUCTION

Recent technological developments have driven new research activity in the field of porous and granular media, permeable membranes. In particular some applications in MEMS require that the characteristics of gas flows through those materials be better understood from the molecular kinetics point of view. In particular one should recall that the gas flow regime in a porous medium can be far from the continuum behaviour when the mean free path takes values comparable with the mean hydraulic diameter of the microchannels in the medium. In this general framework one of the first initial problems was investigated in a rather old paper by Grove and Ford [1]. In short words the problem was: Does a rarefied flow through a porous matrix behave as a bundle of capillary tubes and show characteristics comparable to the single capillary of the reknown Knudsen's experiment [2] ? This means that the mass flow rate through a porous medium should firstly decrease, reach a minimum, and thereafter increase with the pressure drop.

Let a permeability coefficient K_d be defined as in Ref. [3], by

$$K_d = G/\Delta P/L$$

where G (Kg/s) is the mass flow rate and $\Delta P/L$ (Pa/m) is the average pressure gradient in the medium. The definition above was used in Ref.[1] to report the results of some experiments in porous matrices. The results in that paper were judged not conclusive by the authors themselves. But a convincement remained, in the scientific community, that a similar behaviour could be present in tubes and porous media.

In a more recent work than Ref. [1], Kogan *et al*. [4, 5] adopted a theoretical model for a porous medium, and assumed that the matrix can be represented as made of a number of channels of very small width. The flow of a rarefied gas through that system of channels was then studied by means of a MonteCarlo simulation. However the problem of the minimum K_d with the mean pressure drops in the matrix was not addressed in [4, 5].

Another way to simulate and simplify the study of a porous medium is to assume a model corresponding to a bed of spheres [1, 3]. In this case at least the ranges of a few geometrical parameters of the medium such as porosity and tortuosity can be more accurately evaluated.

We tried to give an experimental answer to the question relative to the minimum K_d and adopted a bed of spheres to realize a porous medium of controlled characteristics.

One of the main differences of our experiments, with respect to those already known in literature, is represented by the measurements, at a number of assigned mass flow rates, of the pressure drop distributions along a porous test section. In the past the flow rate corresponding to a total pressure drop or to a mean pressure along the probe was measured. In this last experimental situation the influence of the actual two-dimensional characteristics of the probe was neglected and, in particular, was not considered the influence of the ratio of the probe length to the probe diameter.

To better understand and to complete the information from the experiments we carried out a very simple numerical simulation, where the kinetic model is based on the fact that the gas particles, in a cylindrical bed of spheres, are

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subjected to collisions either with other particles or with the external walls and the surfaces of the spheres. On this point different authors proposed approximate treatments for the two kinds of collisions and an approach similar to the one presented here, but in a quite different situation, was followed in Refs. [6, 7], when dealing with the hypersonic flow against a permeable membrane.

In our case we assume that the occurrence of the two types of collisions, between particles and between particles and solid surfaces, depends upon a probability value β which is related to the geometry of the system and to the characteristics of the flow. β is a parameter to be identified by comparing the experimental results and the results of numerical direct simulations, following a procedure valid for more or less rarefied flow regimes.

EXPERIMENTAL APPARATUS

The experimental probe is a cylindrical stainless steel tube (inner diameter $D = 27 \pm 0.1mm$ and usable length $l = 17 \pm 0.1cm$) filled with randomly close packed glass spheres of three different calibrated diameters $D_p = 0.5, 1, 2(\pm 0.01mm)$. A wide range of pressure drops, between the inlet and the exit of the probe, could be obtained after inserting the cylinder in a test rig where assigned mass flow rates could be realized.

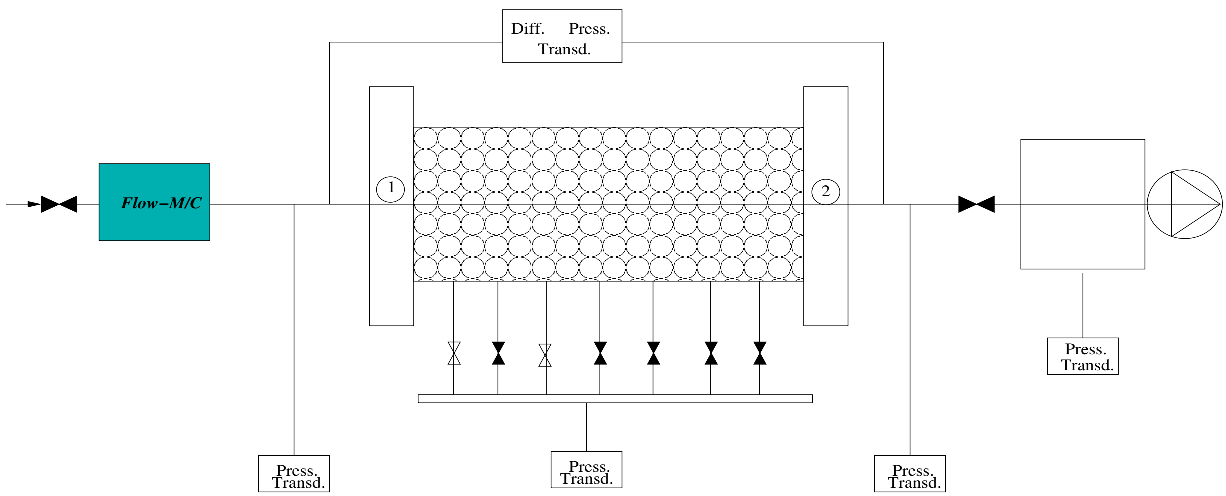

FIGURE 1. Sketch of the experimental set-up.

The test gas is fed through a pressure reduction valve to the flowmeter and to an upstream stagnation chamber. Downstream the probe, the flow goes through a manifold of pin valves to a large stagnation chamber where vacuum is obtained by a Pfeiffer turbomolecular pump. Model J thermocouples are also embedded in the probe. Figure 1 shows a sketch of the experimental apparatus.

Ports along the axis of the test probe enabled the static pressure distribution to be measured. Let x be the distance along the axis of the probe made dimensionless with respect to l. The pressure ports were present at $x = 0.25, 0.5, 0.625, 0.725, 0.875$ and, in addition, the stagnation pressure was measured in the stagnation chambers. In the downstream stagnation chamber a Balzers cold head ion-gauge is present. The test section could be mounted either as in Fig.1 (closed triangles) or by positioning section 2 at the inlet so that the number of measuring stations could be increased (open triangles).

As usually in the vacuum technology, the flow rate was measured in standard cubic centimeters per minute ($1sccm = 1.58\,10^{-5}g/s$). In particular, the mass flow was controlled and measured by a Brook instrument (model 5800) with range $0.1 \div 5\,sccm$ and an accuracy of 2.4 % on minimum reading (rdg).

Pressure measurements were obtained by means of a set of 3 absolute pressure transducers MKS 672 (with range $10^{-4} \div 10^{3}\,mbar$ and an accuracy of 0.12 % of rdg.). A differential pressure transducer MKS Baratron 223 (with range $10^{-3} \div 10\,mbar$ and an accuracy of 0.3 % of rdg.) was also adopted for a further measure of the total pressure drop across the probe.

In all runs Nitrogen was adopted for the experiments, and the permeability coefficient of the beds of spheres as a function of the Reynolds and Knudsen numbers was obtained.

RESULTS

As we already said a Reynolds number Re, representative of the gas motion through the pores, and a significant Knudsen number Kn are the basic parameters for the flow, at each dimensionless probe length x.

For beds of spheres, such as the ones of our experiments, the porosity $\Phi = 0.39$ and the tortuosity factor $\tau \approx 2/3$ are statistically known in literature and are independent of D_p.

In particular the tortuosity factor τ is a parameter associated with one dimensional models of pore structures, and represent the deviation from the macroscopic flow direction of the fluid at every point [3]. It was introduced in Ref.[8] as the ratio between the straight distance connecting the two ends of a capillary tube and the length of the tortuous tube.

They appear in the relation between U_p, the seepage (or interstitial) velocity in the tiny flow channels, i.e. the mean velocity of the gas through the pores, and the Darcy velocity $U = U_p \Phi / \tau$. The mean value U is the ratio between the volume flow rate Q and the cross sectional area of the probe. In particular we take $Re_i = (UL_p/\nu)_i$ and $Kn_i = (\lambda / L_p)_i$, where the subscript i stays for the test section inlet. L_p, ν and λ are the mean pore diameter, the kinematic viscosity and the molecular mean free path.

As an introduction to the following detailed description of the results we show, in Fig.2, the local value of Kn *vs* x at different D_p and Q. We note that the rarefaction effects become more and more evident as one moves toward the outlet, and depend on both D_p and Q. This fact was also verified through the two-dimensional MonteCarlo simulations which will be presented later.

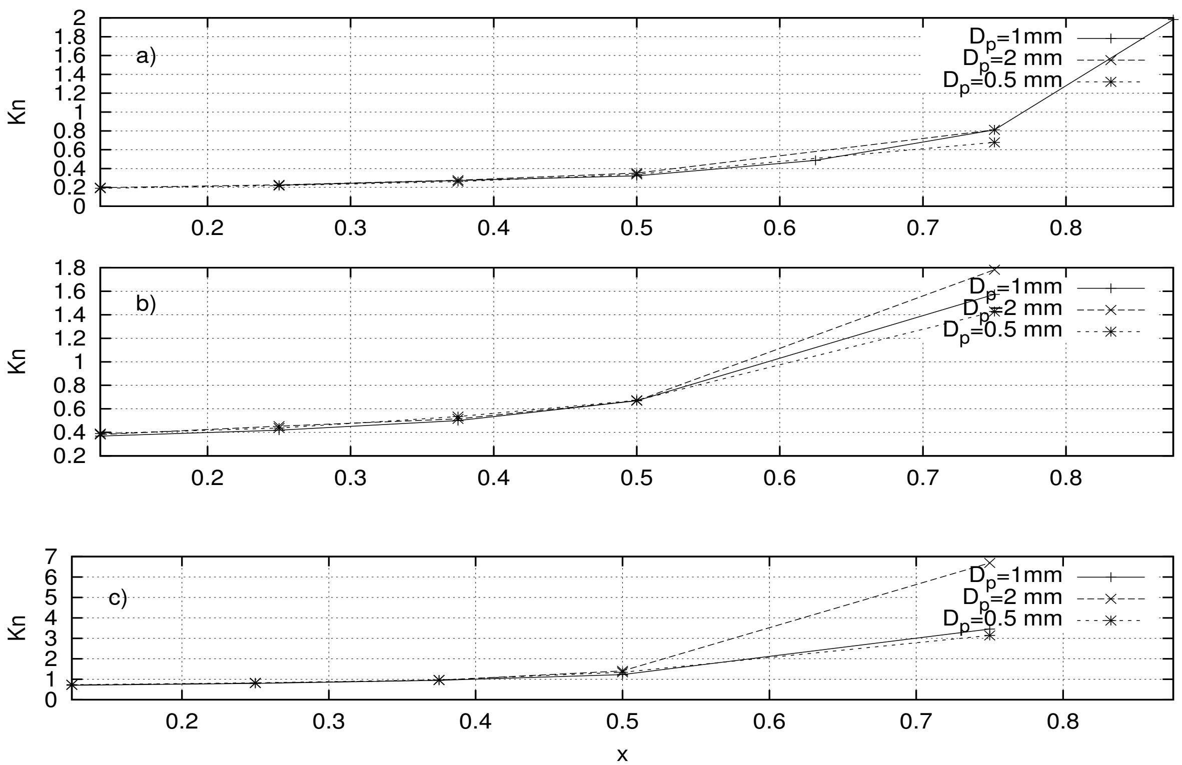

FIGURE 2. Kn distributions *vs* x at different D_p and Q. a) $Q = 1\,sccm$, b) $Q = 0.5\,sccm$, c) $Q = 0.25\,sccm$

Figure 3 presents a typical distribution of pressure along the probe at $Re_i = 6.5\,10^{-4}$ and $Kn_i = 1.05$. Note that the experimental data for the static pressure were taken at the wall. In the same graph the results of a direct simulation which was carried out according to the procedure described in the sequel are also shown. For given Φ and τ, as in our experiments, and after a preliminary evaluation of the influence of the characteristic products, we assume that β is simply a function of $A = Re\,Kn$. Then, to evaluate $\beta(A)$ we run a code based on a DSMC (Direct Simulation MonteCarlo method) for a two dimensional axisymmetric flow of nitrogen through a bed of spheres in a cylindrical

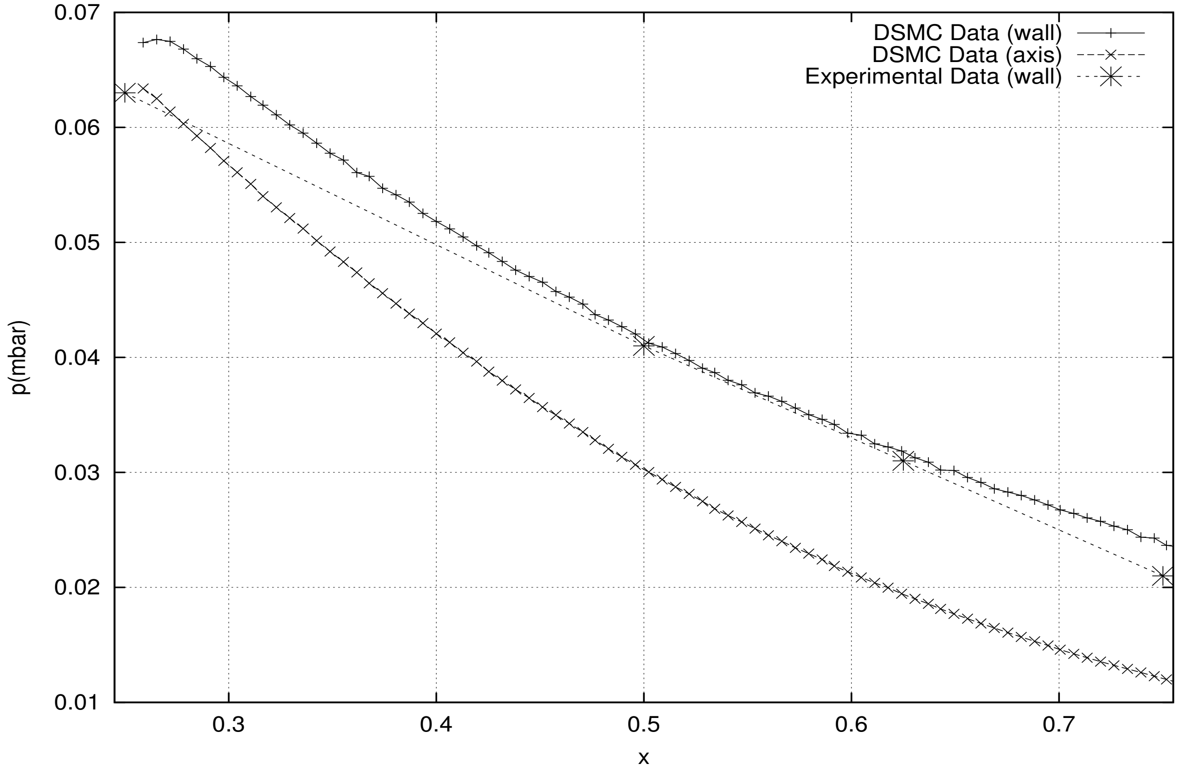

FIGURE 3. Experimental and simulated pressure distributions *vs* x. $Kn_i = 1.05$, $Q = 0.25$, *sccm*

domain of length l and diameter D, where N representative particles are made to collide, at each calculation step, in a cell of the domain, either with other particles or with solid surfaces as the spheres and the walls, according to the assigned β value. Thus, a number of βN gas molecules collide among themselves according to the model of hard spheres whose characteristics for nitrogen can be found in the pertinent literature [9]. Instead the remaining $(1-\beta N)$ molecules hit against the solid which is assumed to re-emit the particles diffusely. Moreover, the velocity distribution function of the re-emitted molecules is assumed to be Maxwellian and corresponding to an assigned temperature distribution in the solid phase whose thermal state is taken independent of the interaction with the gas. In our experiments and in our simulations the temperature of the solid phase was constant and around 296 K.

More experimental results are shown in Figs. 4,5, where the permeability coefficient K_d is given *vs* the mean pressure $p_m = (p_i + p_{i+1})/2$, to be consistent with the data in Ref. [1]. It is noticeable to observe, from the measurement along the probe length, that the permeability parameter K_d as a function p_m shows a very peculiar behaviour when the influence of x is taken into account. We recall that in the majority of our experiments the flow at the exit was choked. Fig.s 4,5 show that, at the high Kn values in the inlet region, K_d decreases with p_m and - after reaching a minimum - increases towards exit.

In particular Fig. 4 reports the minimum squares best fit of the experiments for $x = 0.5, 0.625, 0.75, 0.875$ and Fig. 5 is relative to $x = 0.25$. We note that as the value of x increases, the minimum of function K_d at intermediate p_m tends to become less and less evident.

The DSMC analysis was carried out with 10^6 representative molecules and 100×60 cells. β was determined in each case as we said before and its order of magnitude is the same of A. The comparisons with the experiments are quite reasonable and, above all, show that the influence of the external wall increases with Kn.

The numerical simulations provide the p, Kn and Ma 2D distributions in the medium. For the sake of brevity only two examples of these calculations are given in Fig. 6 where the Ma distributions are shown in two circumstances. The influence of the external wall, increasing with Kn, is clearly shown.

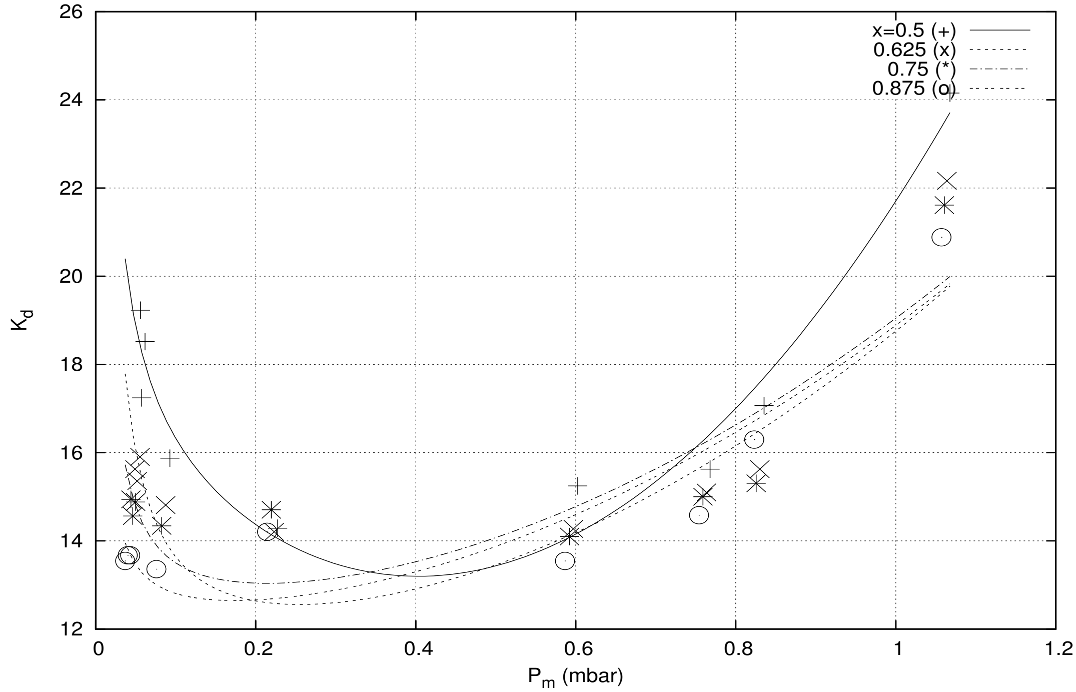

FIGURE 4. Permeability coefficient K_d *vs* p_m, for different values of x.

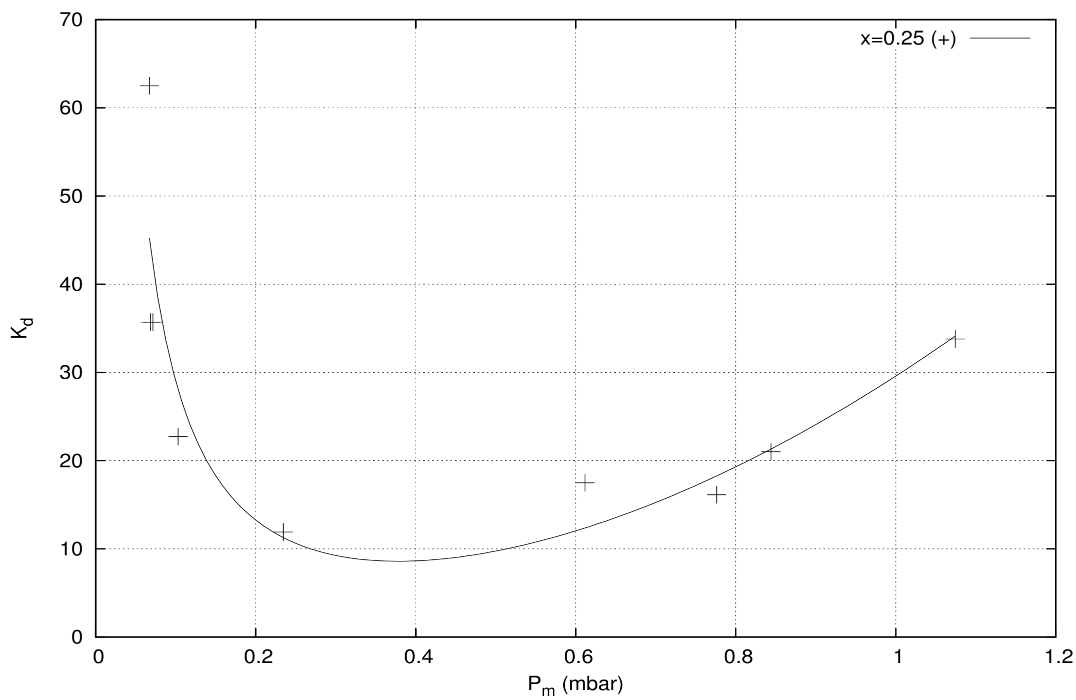

FIGURE 5. Permeability coefficient K_d *vs* p_m, $x = 0.25$

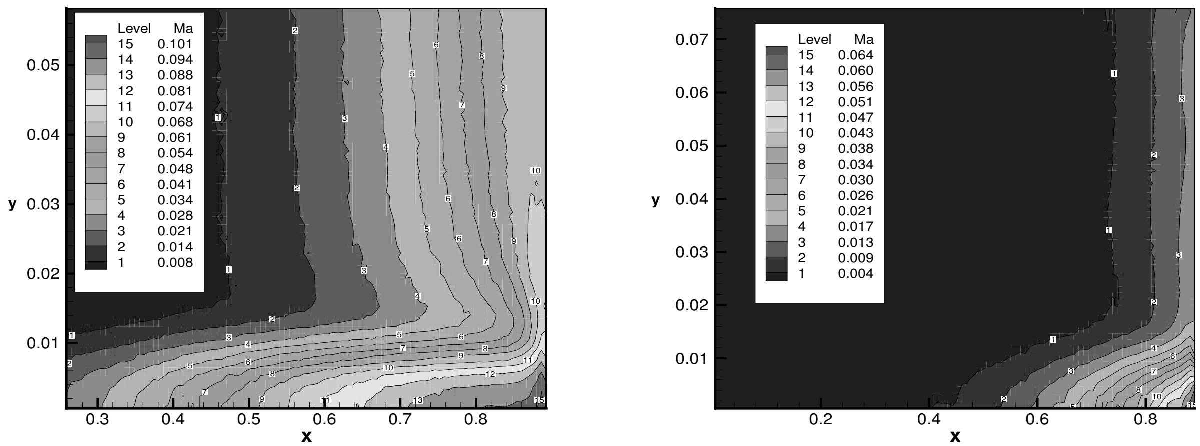

FIGURE 6. Ma distribution. y = dimensionless distance from wall. Left: $Kn_i = 1.05$, $Q = 0.25\,sccm$. Right: $Kn_i = 0.2$, $Q = 1\,sccm$

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